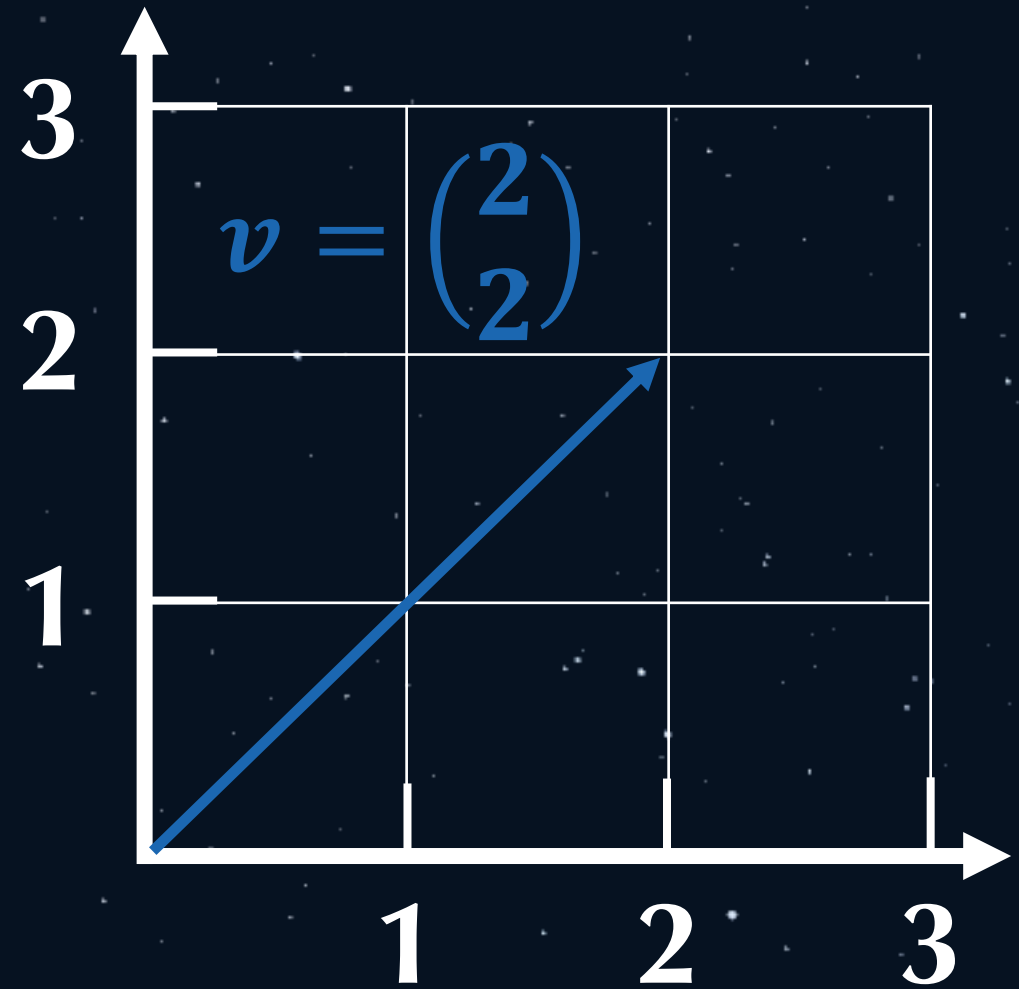
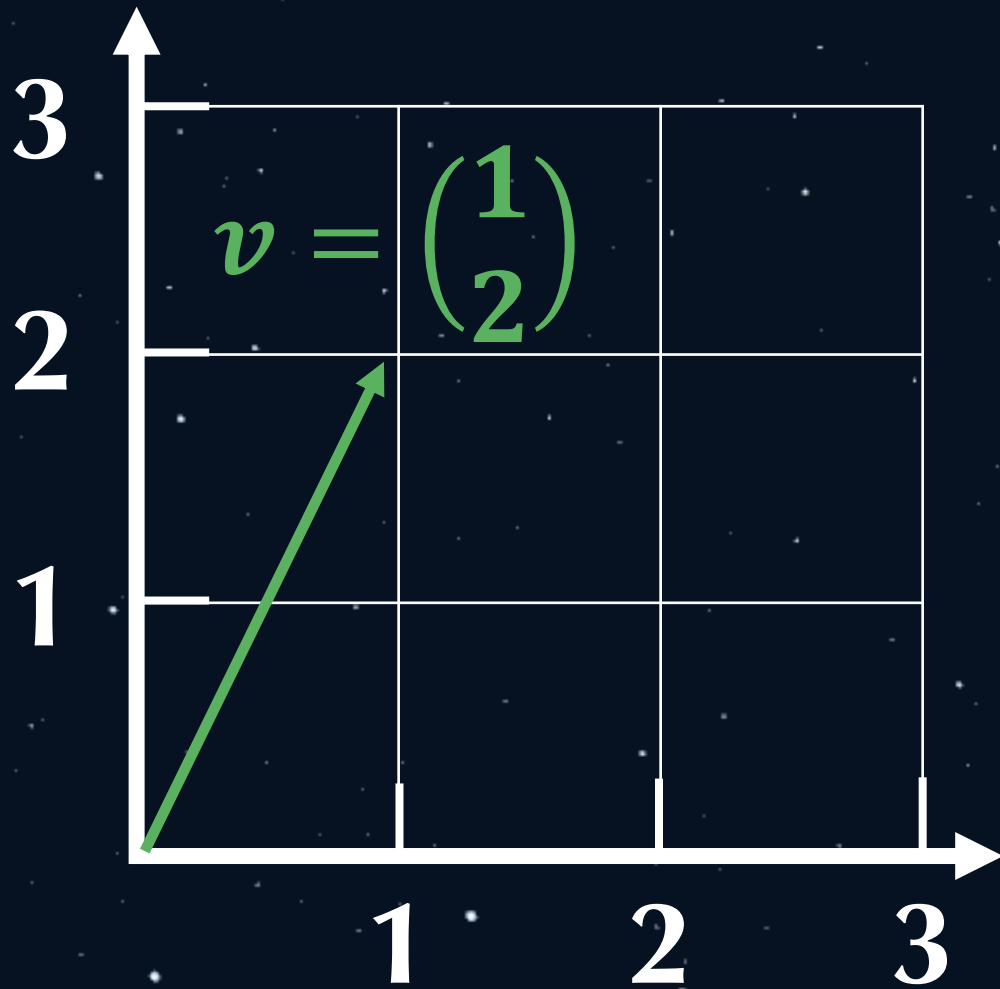
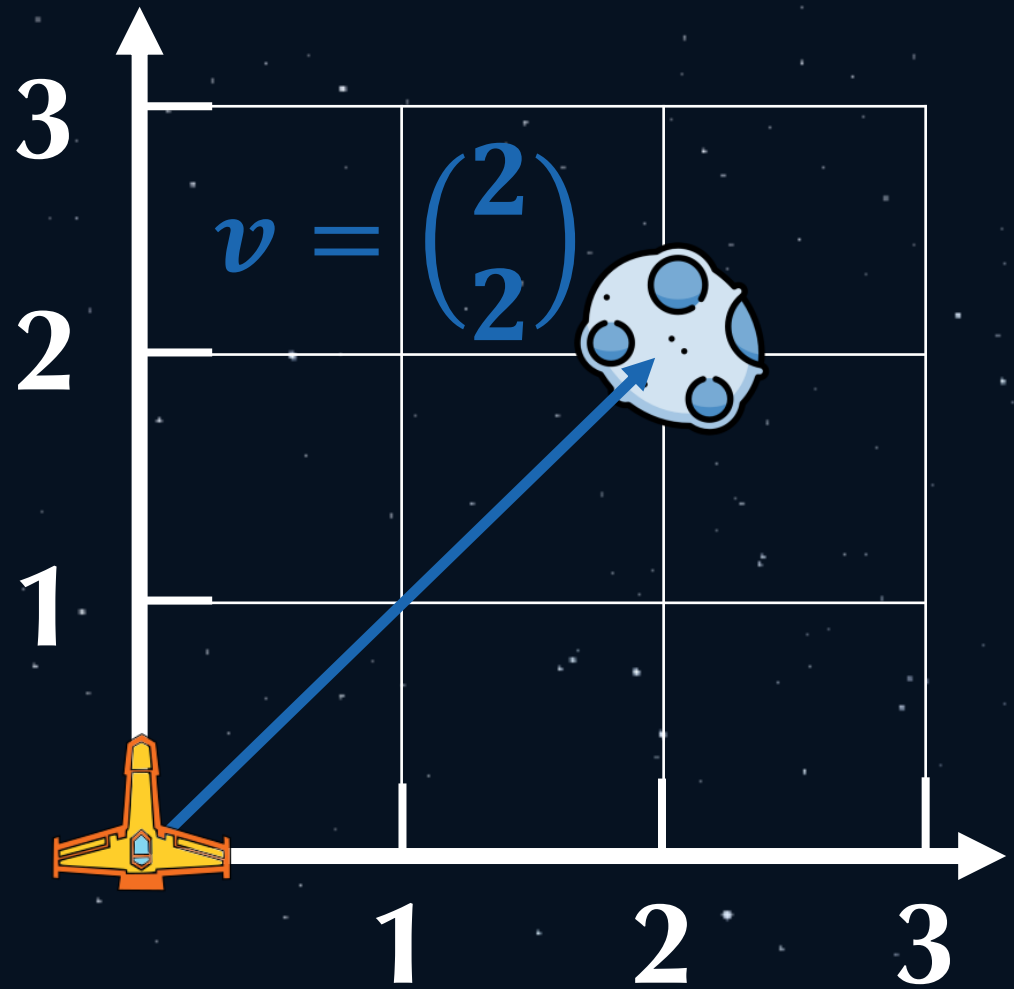
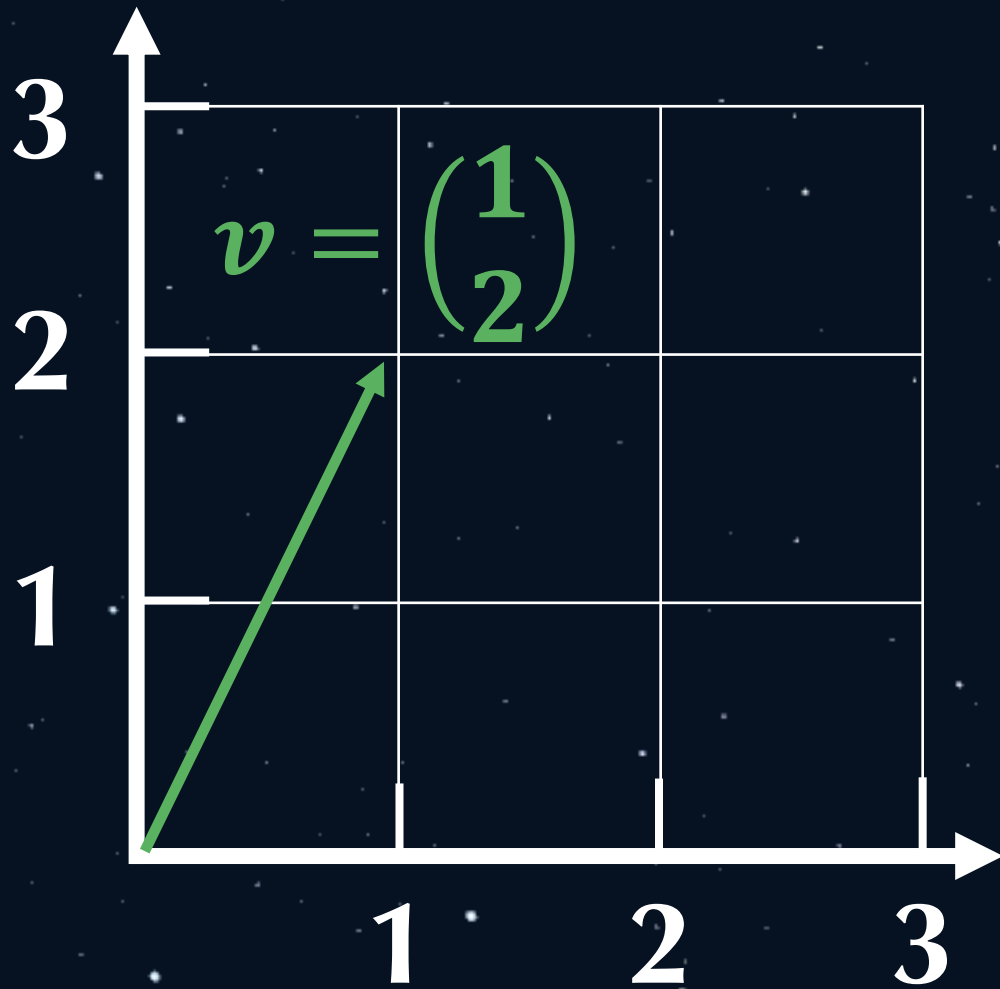


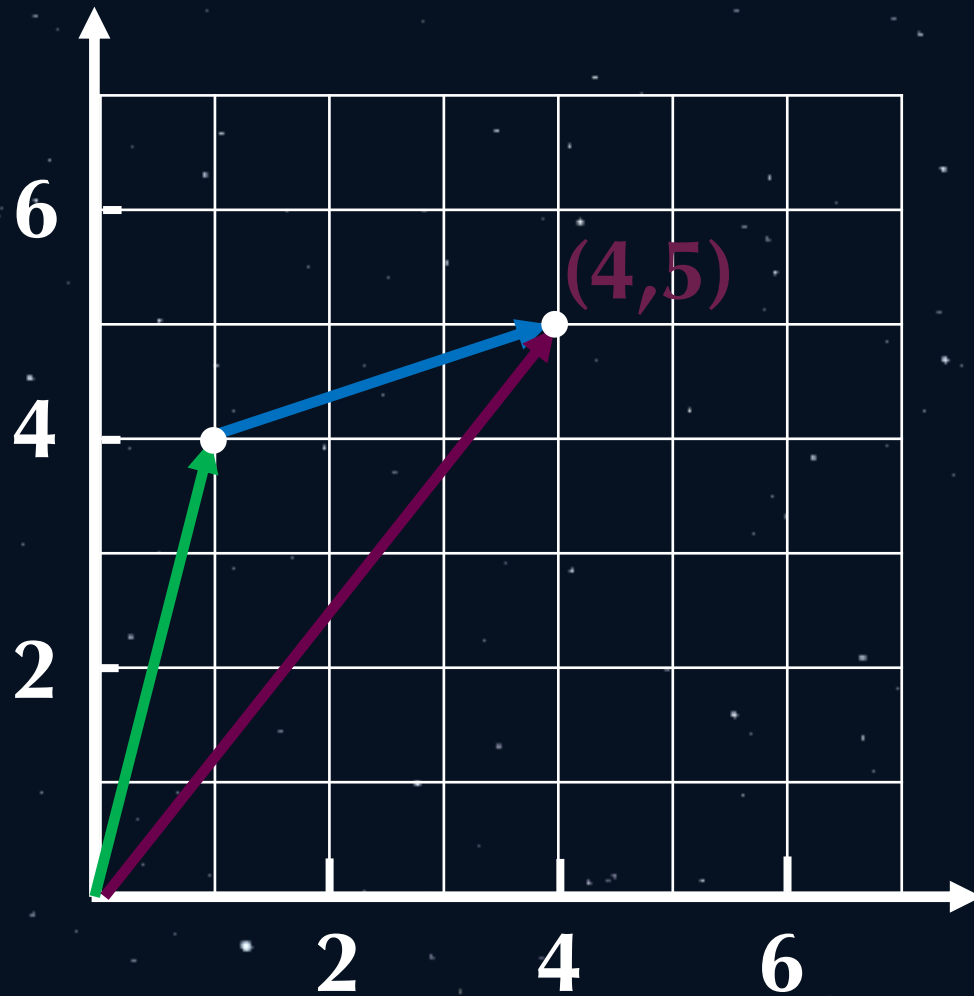
Vectors & Vectors Addition



Vectors are numbers that exist in spaces of more than one dimension. For example, the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ or the vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$. For instance, with vectors, we can describe the distance from the spaceship to the asteroid. The asteroid is located at a distance of two units on the x-axis and two units on the y-axis.



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$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

Adding vectors is very simple. We just have to add the corresponding numbers in the same dimension. That is, we add all the elements corresponding to the x-axis and all the elements corresponding to the y-axis. For example, the sum of the vector $(1\ 4)$ and $(3\ 1)$ is equal to adding $1 + 3 = 4$ and $4 + 1 = 5$. Or in this next example, the sum of the vector $(1\ 4)$ and the vector $(3\ -2)$ is equal to $(4\ 2)$.

Welcome to Starship Voyager!

You are in charge of navigating our starship through hostile space to reach our destination. Follow these instructions carefully to ensure a safe journey.

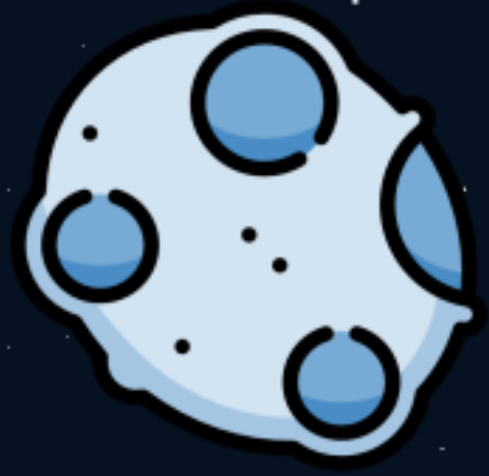
Objective: Your mission is to safely guide the starship to its destination while conserving fuel and avoiding obstacles in space. We are running low on fuel, so efficiency is key. Use the thrusters sparingly and plan your maneuvers wisely to conserve fuel.



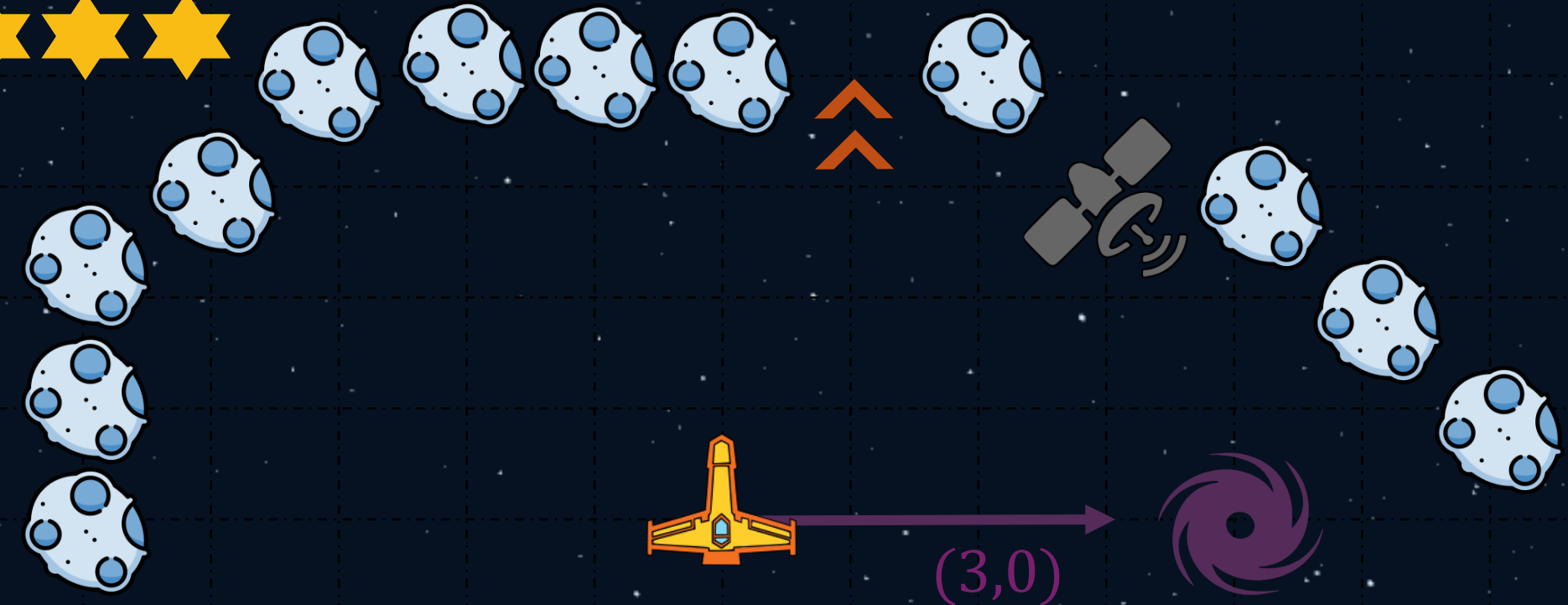
Gravitational Pull: Beware of huge masses in space that can attract the starship due to gravity. Adjust the course of the starship using the directional controls to stay on the planned path and avoid collisions with these gravitational hazards.



Cosmic Winds: Cosmic winds will push the starship off its intended path. Keep an eye on the wind indicators and use the navigation controls to counteract the effect of these winds, ensuring that the starship stays on course.



Obstacle Avoidance: Navigate carefully to avoid satellites and asteroids scattered throughout space. Colliding with these obstacles can damage the starship and jeopardize the mission. Use evasive maneuvers and precise piloting skills to steer clear of danger.

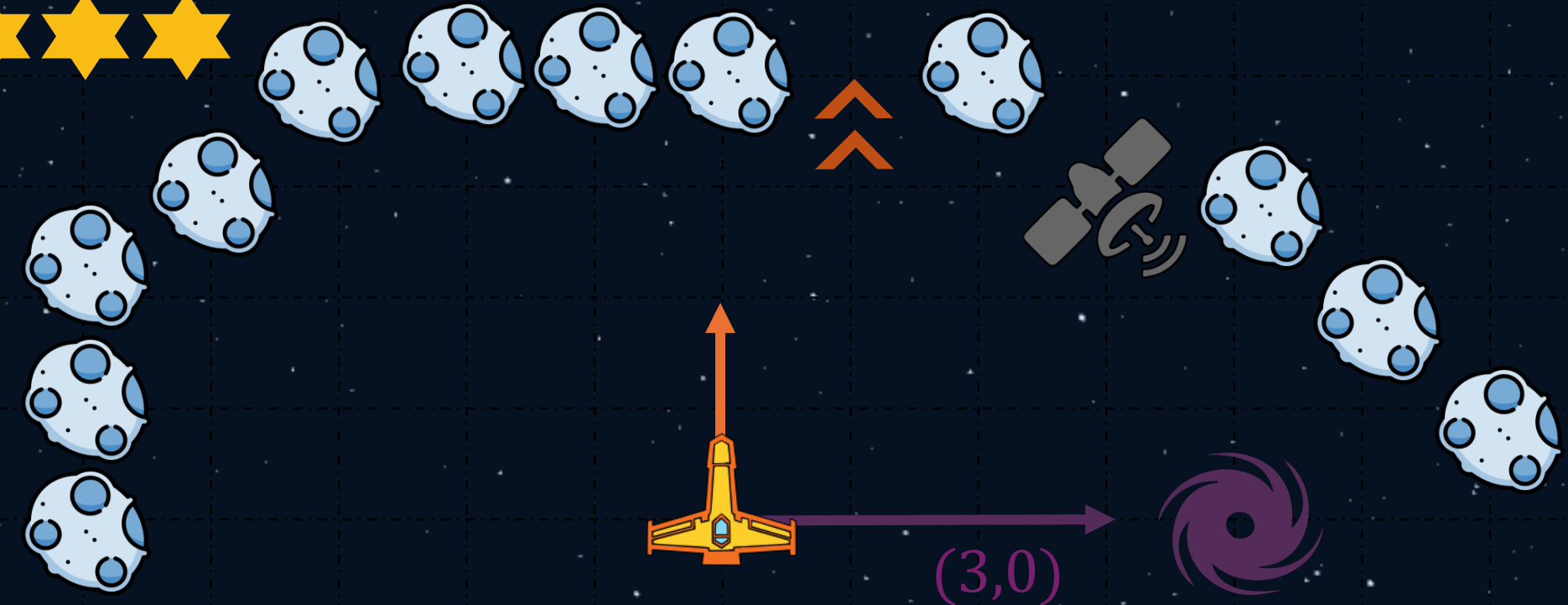


y



X





y

2

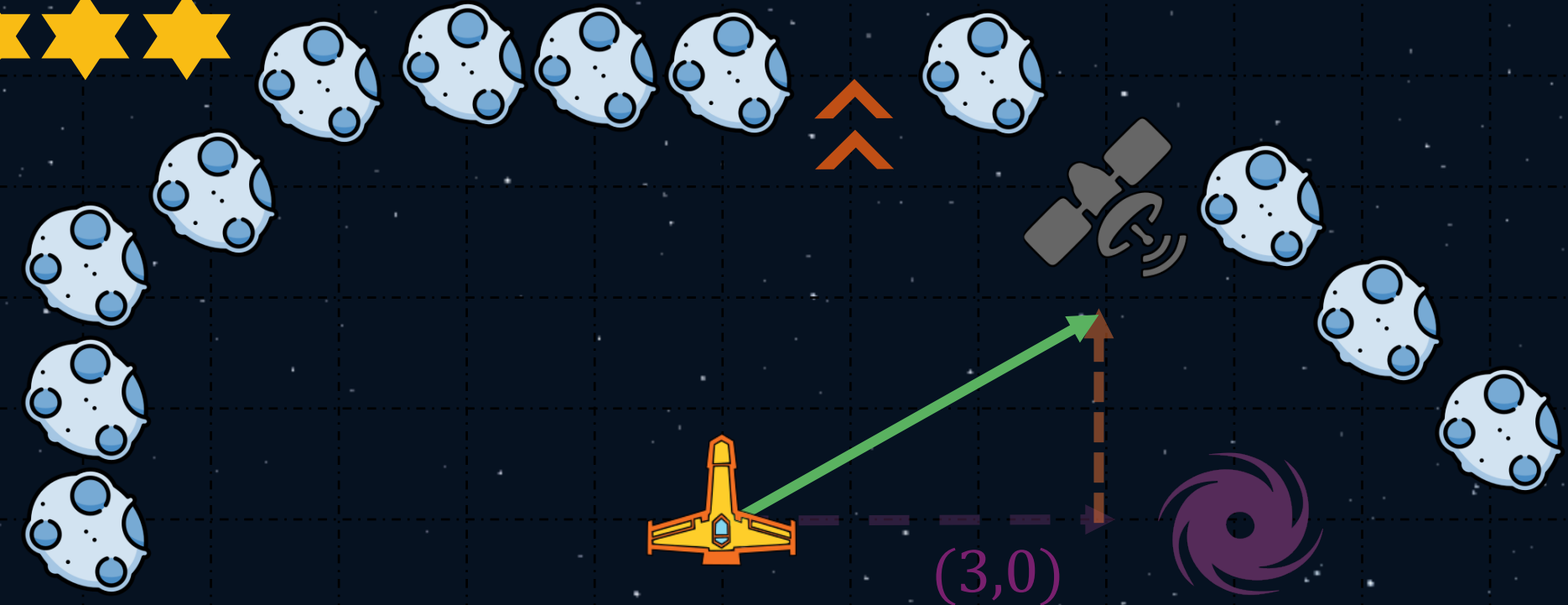


0



X

sum



y

2

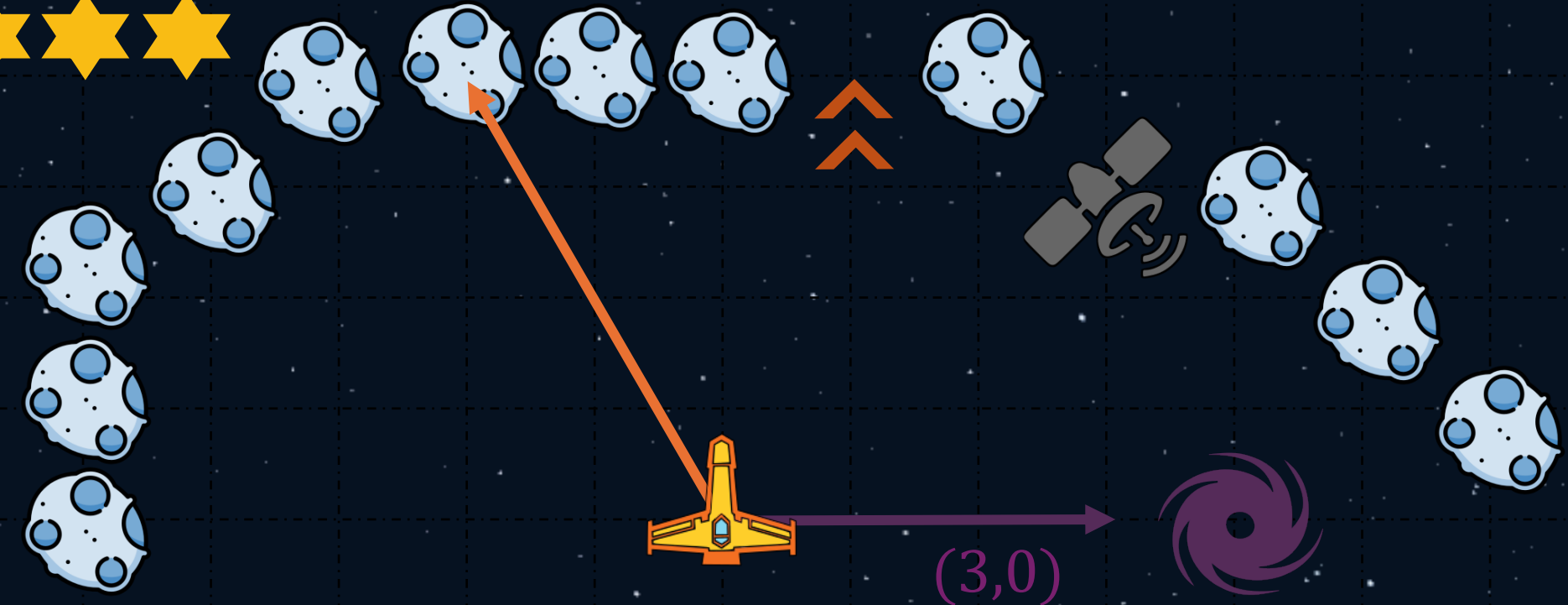


0



x

sum

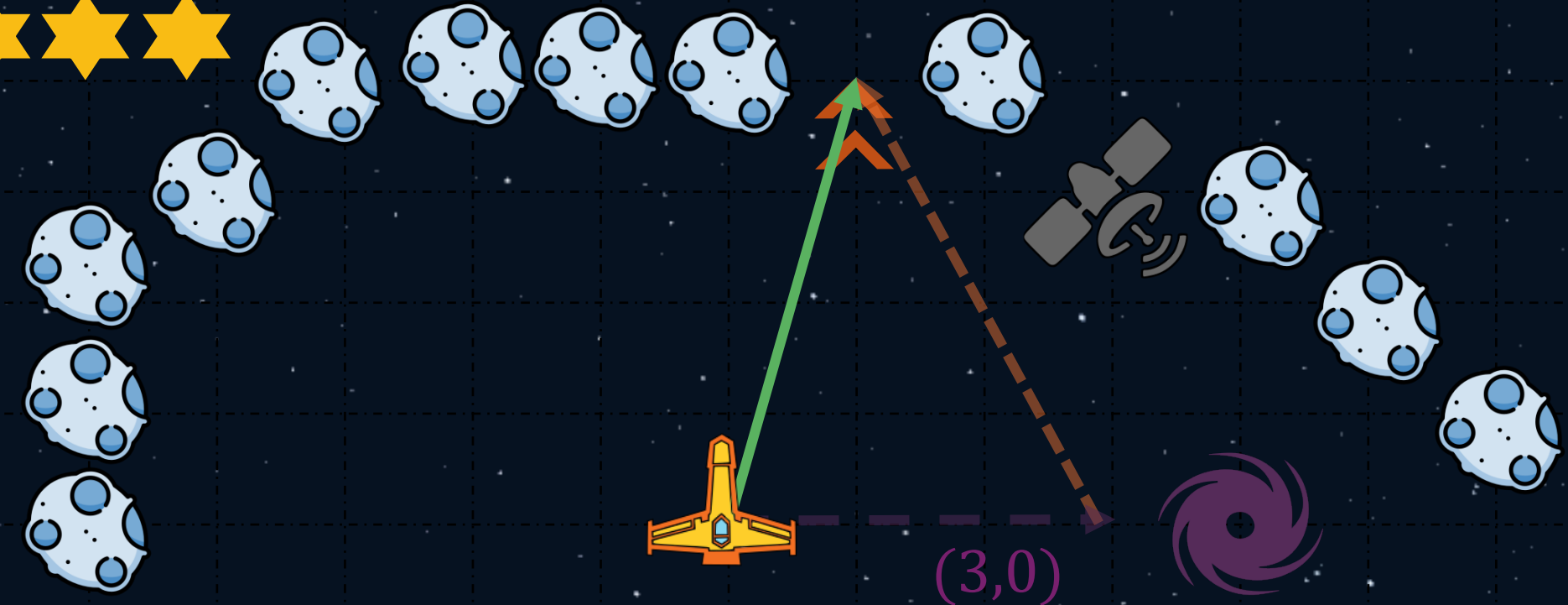


y



x





(3,0)

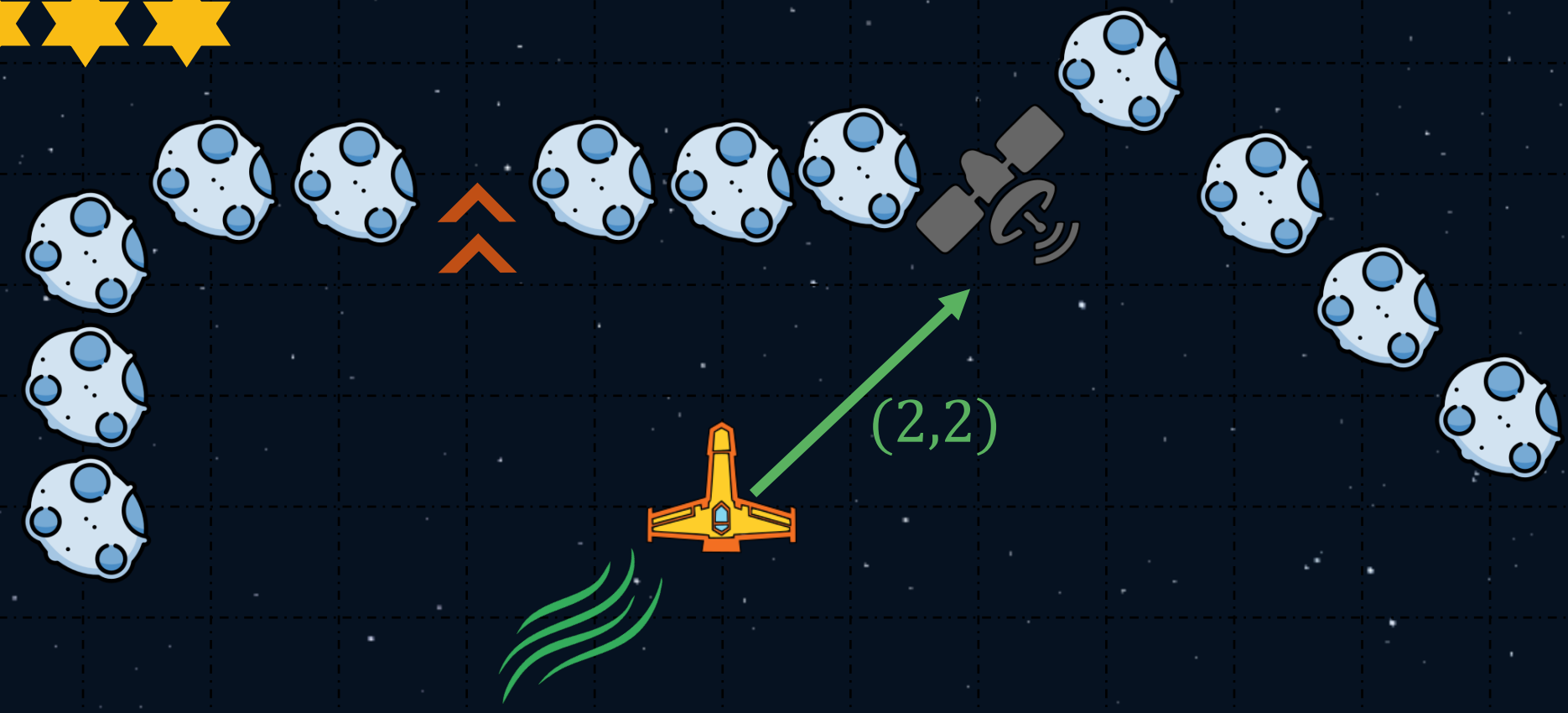
y

4

-2

x

sum



y



X

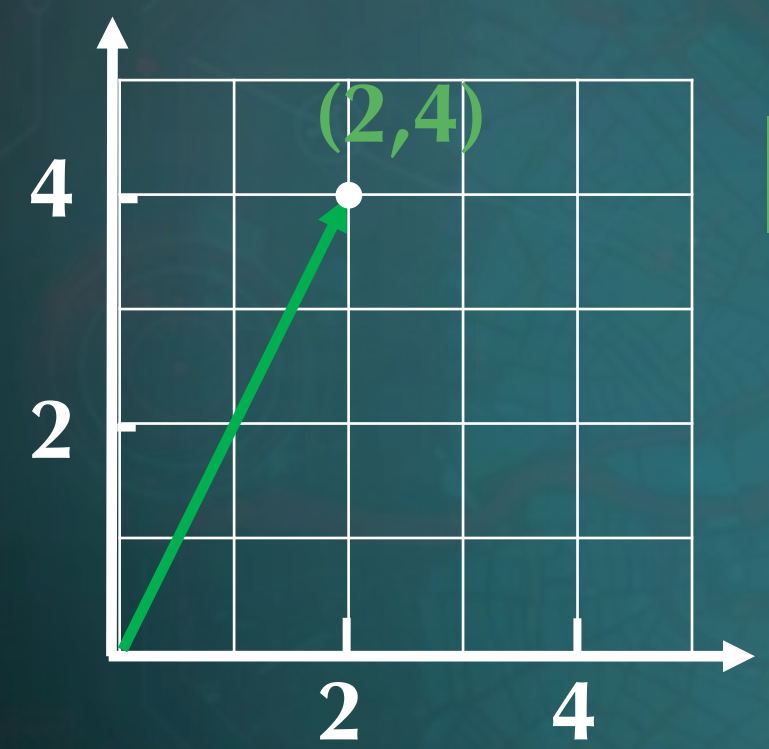


$$||v|| = \sqrt{v_1^2 + v_2^2}$$

Now we will see another vector operation called Norm. The norm is defined as the square sum of all the elements of the vector, and then we take the square root of this sum.

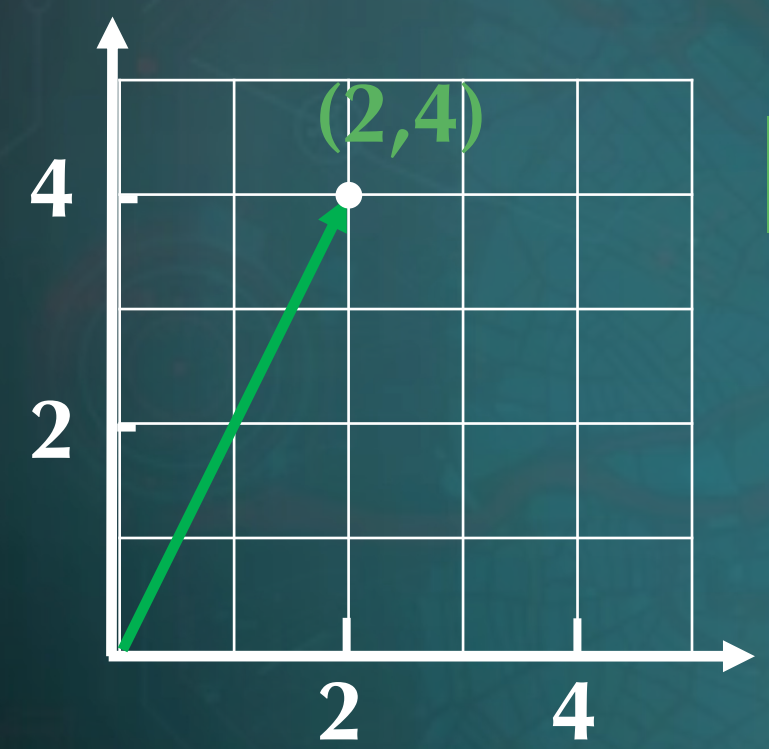
$$\|v\| = \left(\sum_{i=1}^{d=2} |v_i|^2 \right)^{\frac{1}{2}}$$

To be more specific, this is called the Euclidean norm or norm 2. Because as we can see in this variant of the formula, we have a 2 as an exponent, and the square root is expressed as the power of 1/2; which has as denominator 2.



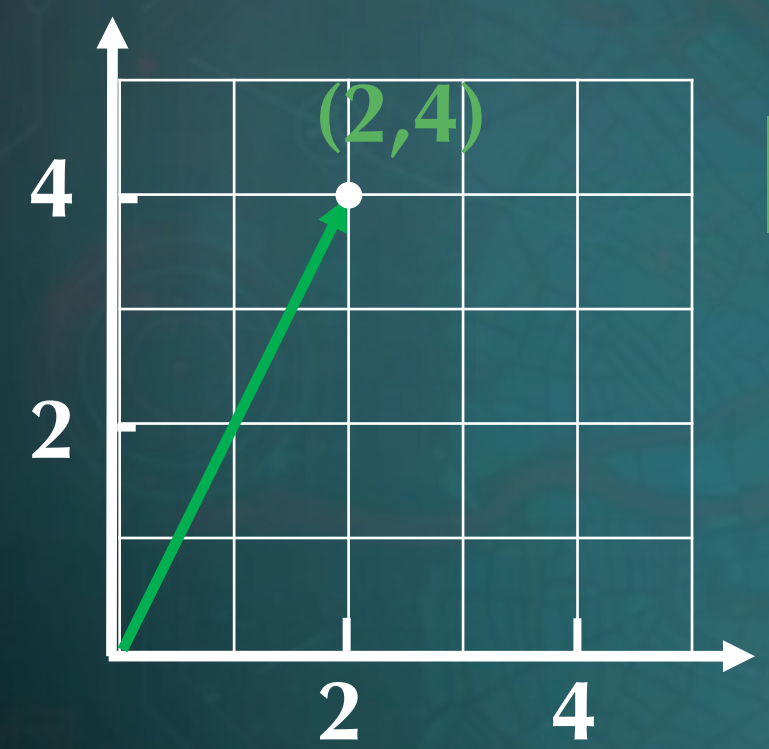
$$\|v\| = \sqrt{2^2 + 4^2}$$

Let's see an example: we have the vector (2,4) and the formula for the norm.



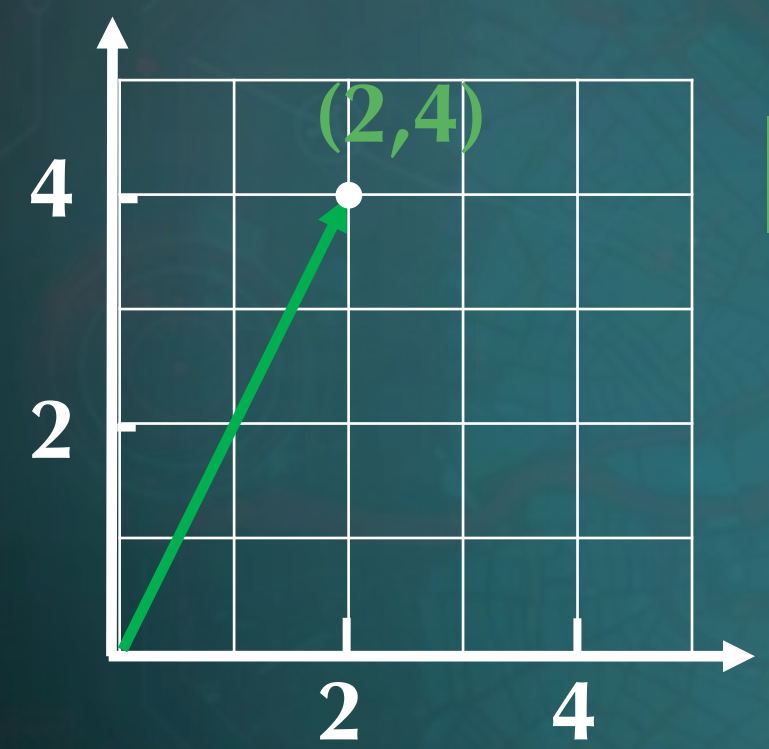
$$\begin{aligned} \|v\| &= \sqrt{2^2 + 4^2} \\ &= \sqrt{4 + 16} \end{aligned}$$

First, we have to square each dimension. In this case, the square of 2 is (2X2=)4 and the square of 4 is (4X4=)16.



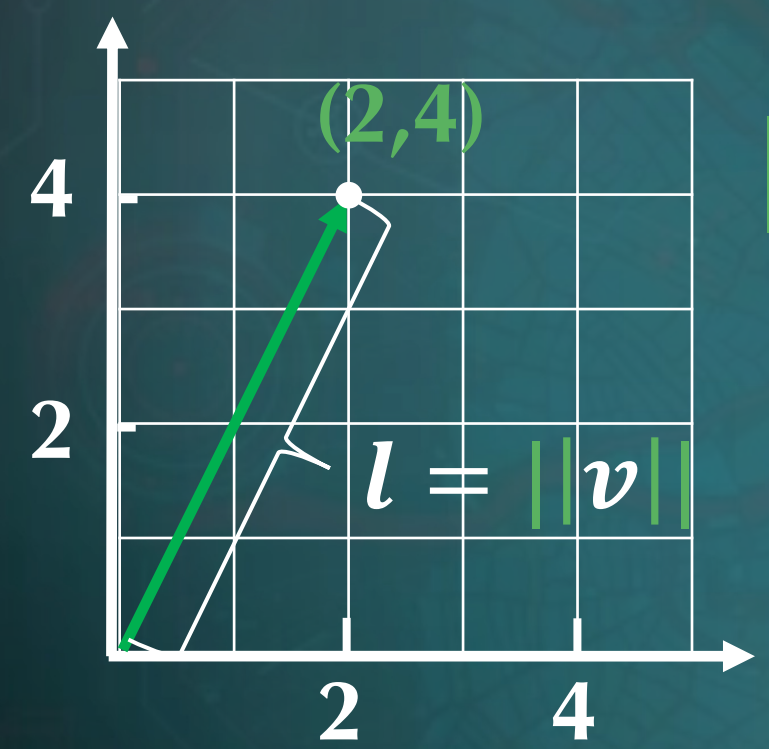
$$\begin{aligned} \|v\| &= \sqrt{2^2 + 4^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \end{aligned}$$

Now we add these two results: $4+16=20$.



$$\begin{aligned} \|v\| &= \sqrt{2^2 + 4^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \\ &= 4.47\dots \end{aligned}$$

Finally, we take the square root of 20, which is 4.47... Therefore, the norm 2 of the vector (2,4) is 4.47...



$$\begin{aligned} ||v|| &= \sqrt{2^2 + 4^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \\ &= 4.47\dots \end{aligned}$$

The Euclidean norm is interpreted as the length of a vector, which is why it is so important.

